Proof of Collatz conjecture

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Abstract. This paper is a trial to prove Collatz conjecture according to the following process.
1. When we have Collatz series of $(n_0, n_1, n_2, \cdots, n_k, n_{k+1}, \cdots, n_{j-1}, n_j)$ by performing Collatz operation for n_0, we find that $n_j < n_0$ holds true after performing Collatz operation a finite number of times for n_0.
2. If Collatz conjecture is true up to $(n_0 - 1)$, Collatz conjecture is true up to n_0 from item 1.
3. It has been confirmed via computer that Collatz conjecture is true up to $2.95 * 10^{20}$.
4. If we regard $2.95*10^{20}$ as (n_0-1), Collatz conjecture is true by mathematical induction.

1. Introduction

1.1 Consider the following operation performed on any positive integer n_0:
If n_0 is even, divide n_0 by 2.
If n_0 is odd, multiply n_0 by 3 and add 1.
Collatz conjecture asserts that if this operation is repeated using the result of the calculation as the new n_0, the result will always reach 1 within a finite number of steps, regardless of the initial value.

1.2 It has been confirmed via computer that Collatz conjecture is true up to $2.95*10^{20}$.

1.3 We define as follows.

1.3.1 Collatz series : When we have the series of $(n_0, n_1, n_2, \cdots, n_k, n_{k+1}, \cdots, n_{j-1}, n_j)$ by performing the following Collatz operation A or Collatz operation B for n_0, we call this series Collatz series. Hereinafter, Collatz series will be abbreviated as CS.
We assume $2.95 * 10^{20} < n_k$. $\quad (k = 0, 1, 2, 3, \cdots\cdots, j-1)$ Because if $n_k \leq 2.95 * 10^{20}$ holds true, this n_k already satisfies Collatz conjecture from item 1.2.

1.3.2 Collatz operation A : If n_k is odd, we have an even number by multiplying n_k by 3 and add 1. Then we have n_{k+1} by dividing the even number by 2. Collatz operation A causes n_k to change into n_{k+1} as shown in the following (1). n_{k+1} can be approximated under the condition of $2.95 * 10^{20} < n_k$. Hereinafter, Collatz operation A will be abbreviated as COA.

$$n_{k+1} = (3 * n_k + 1)/2 = (3/2) * n_k + 1/2 \fallingdotseq (3/2) * n_k \tag{1}$$

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1.3.3 Collatz operation B : If n_k is even, we have n_{k+1} by dividing the even number by 2. Collatz operation B causes n_k to change into n_{k+1} as shown in the following (2). Hereinafter, Collatz operation B will be abbreviated as COB.

$$n_{k+1} = (1/2) * n_k \tag{2}$$

2. Investigation of Collatz operation

2.1 Now we have $n_0(> 2.95 * 10^{20})$. Then we have n_1 by performing COA or COB on n_0 when n_0 is odd or even. Repeating the same task we have CS of $(n_0, n_1, n_2, \cdots, n_k, n_{k+1}, \cdots, n_{j-1}, n_j)$. If there are x odd elements and y even elements in $(n_0, n_1, n_2, \cdots, n_k, n_{k+1}, \cdots, n_{j-1})$, then we have the following (3) from (1) and (2).

$$n_j = (3/2)^x * (1/2)^y * n_0 \qquad (x + y = j) \tag{3}$$

In order for $n_j < n_0$ to hold the following (4) must hold true from (3). We have the following (5) and (6) from (4).

$$(3/2)^x * (1/2)^y < 1 \tag{4}$$
$$x * (\log 3 - \log 2) - y * \log 2 < 0 \tag{5}$$
$$y > x * (\log 3/\log 2 - 1) = 0.58 * x \tag{6}$$

2.2 From (1) we can find that if n_k is odd number of $\{2 * (\text{odd number}) + 1\}$ type, n_{k+1} can be an odd number and if n_k is odd number of $\{2 * (\text{even number}) + 1\}$ type, n_{k+1} can be an even number. Since the two types of odd numbers appear alternately, there is 50% probability of n_{k+1} being odd or even.
Similarly from (2) we can find that if n_k is an even number of 2∗(odd number) type, n_{k+1} can be an odd number and if n_k is even number of 2∗(even number) type, n_{k+1} can be even number. Since the two types of even numbers appear alternately, there is 50% probability of n_{k+1} being odd or even.
Therefore the probability of each element of $(n_0, n_1, n_2, \cdots, n_k, n_{k+1}, \cdots, n_{j-1})$ being odd or even is 50%. Then we have the following (7), (8), (9) from (3), (7), (8) respectively.

$$j = x + y \quad \sim \quad j/2 + j/2 \qquad (j \to \infty) \tag{7}$$
$$n_j = (3/2)^x * (1/2)^y * n_0 \quad \sim \quad (3/2)^{j/2} * (1/2)^{j/2} * n_0 = (3/4)^{j/2} * n_0 \qquad (j \to \infty) \tag{8}$$
$$\lim_{j \to \infty} n_j = 1 \tag{9}$$

The above (9) shows that n_j approaches 1 with $j \to \infty$. In other words, $n_j(< n_0)$ can be obtained through a finite number of COA or COB for n_0.

2.3 Here we imagine the odd-first CS where the first $r(\geq 2)$ consecutive elements are odd numbers like the following (10). Please refer to the following item 2.4 for detail of this odd-first CS.

$$\underbrace{(n_0, n_1, n_2, \cdots, n_{r-1},}_{r \text{ elements : odd number}} n_r, \cdots, n_{j-1}, n_j) \tag{10}$$

In the following (Figure 1) the yellow area shows $0.58 * x < y$. (x, y) of CS exists in this yellow area after a finite number of Colatz operation for n_0 from (6). Plotting (x, y) for the above odd-first CS from n_0 to n_k results in the red line. $(k = 0, 1, 2, 3, \cdots\cdots, j-1, j)$ From $k = 0$ to $k = r-1$, y is always zero and x increases by 1 with increase of k. From $k = r$ to $k = j$, x and y increase with 50% probability as shown in item 2.2. Since the red line is a curve with an average rate of increase of 1 in $r \leq k$, the line $y = 0.58 * x$ and the red line always have one intersection. Then in the odd-first CS $n_j < n_0$ holds true after performing COA or COB a finite number of times for n_0.

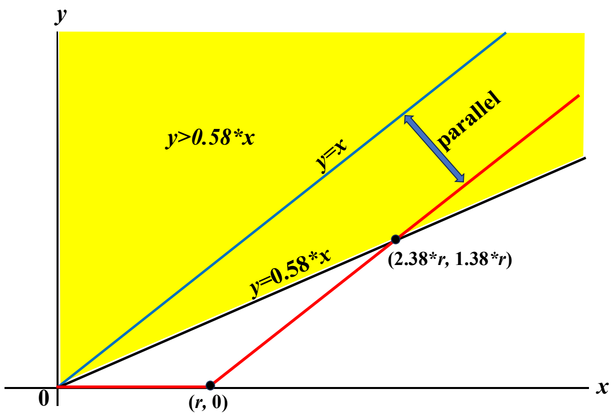

Figure 1 : (x, y) of the odd-first CS

2.4 When we have $n_0 = 2^r * n + 2^r - 1$, we have $(n_1, n_2, n_3, \cdots\cdots, n_r)$ by COA as follows. $(r, n$: natural number)

$$\begin{aligned}
n_0 &= 2^r * n + 2^r - 1 && : \text{odd number} \\
n_1 &= 3 * 2^{r-1} * n + 3 * 2^{r-1} - 1 && : \text{odd number} \\
n_2 &= 3^2 * 2^{r-2} * n + 3^2 * 2^{r-2} - 1 && : \text{odd number} \\
n_3 &= 3^3 * 2^{r-3} * n + 3^3 * 2^{r-3} - 1 && : \text{odd number} \\
&\vdots \\
n_{r-2} &= 3^{r-2} * 2^2 * n + 3^{r-2} * 2^2 - 1 && : \text{odd number} \\
n_{r-1} &= 3^{r-1} * 2 * n + 3^{r-1} * 2 - 1 && : \text{odd number} \\
n_r &= 3^r * n + 3^r - 1 && : \text{odd or even number}
\end{aligned}$$

As shown above there exists n_0 that continues to increase by COA, but eventually, it stops increasing and becomes the state of item 2.2.

3. Investigation of Collatz series

3.1 The following (Figure 2) is flow chart for checking $n_j < n_0$.

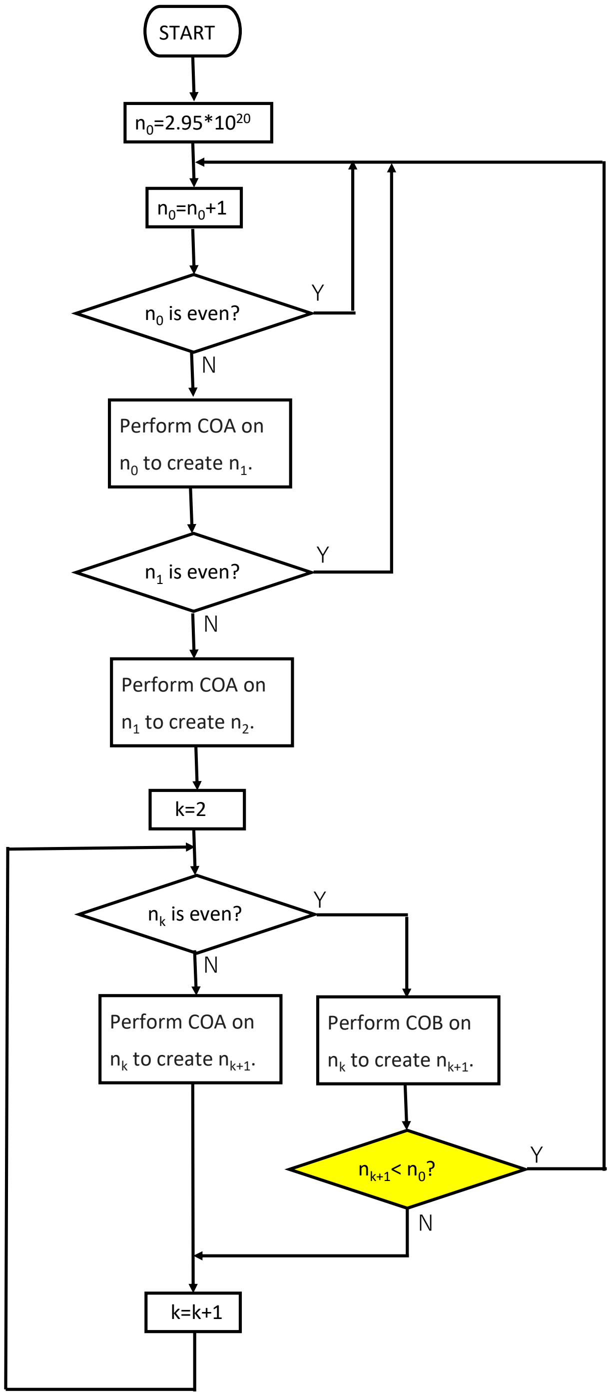

Figure 2 : Flow chart for checking $n_j < n_0$

3.2 It has been confirmed via computer that Collatz conjecture holds for n_0 up to $2.95 * 10^{20}$. We will start from $n_0 = 2.95 * 10^{20} + 1$.
If n_0 is even, the check of n_0 is completed by $n_1 = n_0/2 < n_0$ and the check moves to $n_0 = 2.95 * 10^{20} + 2$.
If n_0 is odd and n_1 is even, the check of n_0 is completed by $n_2 = (3/2) * (1/2) * n_0 = (3/4) * n_0 < n_0$ and the check moves to $n_0 = 2.95 * 10^{20} + 2$.

3.3 If both n_0 and n_1 are odd, this CS is the odd-first CS where the first $r(\geq 2)$ consecutive elements are odd numbers like the above (10). As shown in item 2.3, in the odd-first CS $n_j < n_0$ holds true after performing COA or COB a finite number of times for n_0. Then the check of n_0 is completed and the check moves to $n_0 = 2.95 * 10^{20} + 2$.

3.4 If we repeat the operation of item 3.1—3.3, we can confirm that $n_j < n_0$ holds true in $n_0 = 2.95 * 10^{20} + 1, 2.95 * 10^{20} + 2, 2.95 * 10^{20} + 3, \cdots\cdots$.

4. Conclusion

Collatz conjecture is true due to the following reasons.

4.1 When we have CS of $(n_0, n_1, n_2, \cdots, n_k, n_{k+1}, \cdots, n_{j-1}, n_j)$ by performing COA or COB for n_0 and there are x odd elements and y even elements in $(n_0, n_1, n_2, \cdots, n_k, n_{k+1}, \cdots, n_{j-1})$, $n_j < n_0$ holds true under the condition of $0.58 < y/x$.

4.2 Since the probability of each element of $(n_0, n_1, n_2, \cdots, n_k, n_{k+1}, \cdots, n_{j-1})$ being odd or even is 50%, y/x approaches to 1 with $j \to \infty$.

4.3 $n_j < n_0$ holds true after performing COA or COB a finite number of times for n_0 from item 4.1 and 4.2.

4.4 If Collatz conjecture is true up to $(n_0 - 1)$, Collatz conjecture is true up to n_0 from item 4.3. Because if we regard $n_j(\leq n_0 - 1)$ as the new n_0, this new n_0 satisfies Collatz conjecture.

4.5 It has been confirmed via computer that Collatz conjecture is true up to $2.95 * 10^{20}$.

4.6 If we regard $2.95 * 10^{20}$ as $(n_0 - 1)$, Collatz conjecture is true by mathematical induction.

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