Proof of Fermat's Last Theorem

By Toshihiko Ishiwata

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Abstract. This paper is a trial to prove Fermat's Last Theorem according to the following process.

1. Fermat's Last Theorem claims that the following (1) has no solution of (x, y, z).

$$X^n + Y^n = Z^n \qquad (3 \leq n \quad X, Y, Z, n : \text{natural number}) \qquad (1)$$

2. We find that the above (1) does not have the solution in $n_1 \leq n$ from the total number of pair (x, y) which may satisfy (1) at fixed z and the probability that a natural number close to z^n is a power of n.
3. It is confirmed by computers by the early 1990s that (1) has no solution from $n = 3$ up to $n = 4 * 10^6$.
4. We find that $n_1 = 4$ holds true from item 2. and 3.
5. Fermat's Last Theorem is true from item 3. and 4.

1. Introduction

1.1 Fermat's Last Theorem claims that the following (1) has no solution of (x, y, z).

$$X^n + Y^n = Z^n \qquad (3 \leq n \quad X, Y, Z, n : \text{natural number}) \qquad (1)$$

If (1) has the solution of (x, y, z), $(l * x, l * y, l * z)$ can also be the solution of (1). $(l = 2, 3, 4, \cdots\cdots)$ Then the total number of solutions of (1) is always zero or ∞ at any n. Since this is inconvenient, we define the following (2). We also define the following (3) from the following item 1.2 and 1.3.

$$gcd(x, y, z) = 1 \qquad (2)$$

$$1 \leq x < y < z \qquad (3)$$

1.2 If $x = y$ holds true, the above (1) has no solution as shown in the following (4). Because $2^{1/n}$ is an irrational number. Then we define $x \neq y$ as shown in the above (3).

$$x^n + y^n = 2 * x^n = z^n \qquad 2^{1/n} * x = z \qquad (x = y) \qquad (4)$$

1.3 If (1) has the solution of $(x, y, z) = (a, b, c)$, $(x, y, z) = (b, a, c)$ can also be the solution. Then we define $x < y$ in the above (3). Even with this definition, the claim of Fermat's Last Theorem remains unchanged.

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1.4 It is confirmed by computers by the early 1990s that (1) has no solution from $n = 3$ up to $n = 4 * 10^6$.

2. The total number of solutions of (1)

The total number of solutions of (1) decreases with the increase of n and becomes zero in $n_1 \leq n$ due to the following reason.

2.1 There are $A = \lfloor t^{1/n} \rfloor \fallingdotseq t^{1/n}$ powers of n between 1 and the natural number t. Then the probability $P(t)$ that t is a power of n can be expressed by the following (5). Because $P(t)$ is the rate of increase of A with respect to t.

$$P(t) = \frac{dA}{dt} = \frac{t^{1/n-1}}{n} \tag{5}$$

The following (6) represents the probability that a natural number close to $t = z^n$ is a power of n from (5).

$$P(z^n) = \frac{z^{1-n}}{n} \tag{6}$$

2.2 If (1) has the solution of (x, y, z), we have the following (7) from (1).

$$x^n + y^n = z^n \tag{7}$$

We have the following (8) from (3) and (7). We have the following (9) from (8).

$$1 \leq x^n < \frac{x^n + y^n}{2} = \frac{z^n}{2} < y^n < z^n \tag{8}$$

$$1 \leq x < \frac{z}{2^{1/n}} < y < z \tag{9}$$

We have the following (10) from (9). And we have the following (11) from (10).

$$\begin{aligned} &\{\text{the total number of } x\} = \lfloor \frac{z}{2^{1/n}} \rfloor \fallingdotseq \frac{z}{2^{1/n}} \\ &\{\text{the total number of } y\} = z - \lfloor \frac{z}{2^{1/n}} \rfloor \fallingdotseq z - \frac{z}{2^{1/n}} \end{aligned} \tag{10}$$

$$\{\text{the total number of } x\} - \{\text{the total number of } y\} \fallingdotseq (2^{1-1/n} - 1) * z > 0 \quad (3 \leq n) \tag{11}$$

2.3 Since {the total number of x} is larger than {the total number of y} from (11), the total number of pair (x, y) which may satisfy (7) at fixed z is equal to {the total number of y}. Because when both y and z are fixed, there is exactly one x that is closest to $(z^n - y^n)^{1/n}$. This x may satisfy (7) along with the fixed y and z. The second closest x to $(z^n - y^n)^{1/n}$ can never satisfy (7).
Then the total number of the solution which satisfies (7) at the fixed z can be expressed by the following (12) from (6) and (10).

$$P(z^n) * \{\text{the total number of } y\} = \frac{z^{1-n}}{n} * (z - \frac{z}{2^{1/n}}) = (1 - \frac{1}{2^{1/n}}) * \frac{z^{2-n}}{n} \tag{12}$$

Therefore $N(n)$: {the total number of the solution which satisfies (7) at n} can be expressed by the following (13) from (12).
Since we can find $2 \leq y$ and $3 \leq z$ from (3), we calculate $N(n)$ by integrating (12) from $z = 3$ to $z = \infty$.

$$\begin{aligned} N(n) =& \lfloor (1 - \frac{1}{2^{1/n}}) * \frac{1}{n} * \int_3^\infty z^{2-n}\, dz \rfloor = \lfloor (1 - \frac{1}{2^{1/n}}) * \frac{1}{n(3-n)} * \left[z^{3-n} \right]_3^\infty \rfloor \\ =& \lfloor (1 - \frac{1}{2^{1/n}}) * \frac{3^{3-n}}{n(n-3)} \rfloor \qquad (4 \leq n) \qquad (13) \end{aligned}$$

At $n = 3$ the following integral value in (13) diverges to ∞ with $z \to \infty$.

$$\int_3^\infty z^{2-n}\, dz = \int_3^\infty z^{-1}\, dz = [\log z]_3^\infty \qquad (n = 3)$$

Then $N(n)$ converges in $4 \leq n$.

2.4 We have the following (14) from (13). Because both $(1 - \frac{1}{2^{1/n}})$ and $\frac{3^{3-n}}{n(n-3)}$ decrease with the increase of n and converge to zero with $n \to \infty$.

$$\lim_{n \to \infty} N(n) = 0 \qquad (14)$$

$N(n)$ decreases with the increase of n and becomes zero in $n_1 \leq n$ from the above (14). Because $N(n)$ can have only the value of an integer.

3. Investigation of n_1

3.1 The range of n_1 is limited to the following 3 cases.
Case 1 : $n_0 = 4 * 10^6 < n_1$
Case 2 : $4 < n_1 \leq n_0$
Case 3 : $n_1 = 4$
3 cases are shown in the following (Figure 1) in the next page.

3.2 Case 1 and Case 2 cannot exist because they contradict item 1.4. Then we can find that Case 3 shows the real n_1. We have the following (15).

$$N(4) = \lfloor (1 - 2^{-1/4})/12 \rfloor = \lfloor 0.013 \rfloor = 0 \qquad (15)$$

Then (1) has no solution in $4 \leq n$.

4. Conclusion

Fermat's Last Theorem is true from item 1.4 and 3.2.

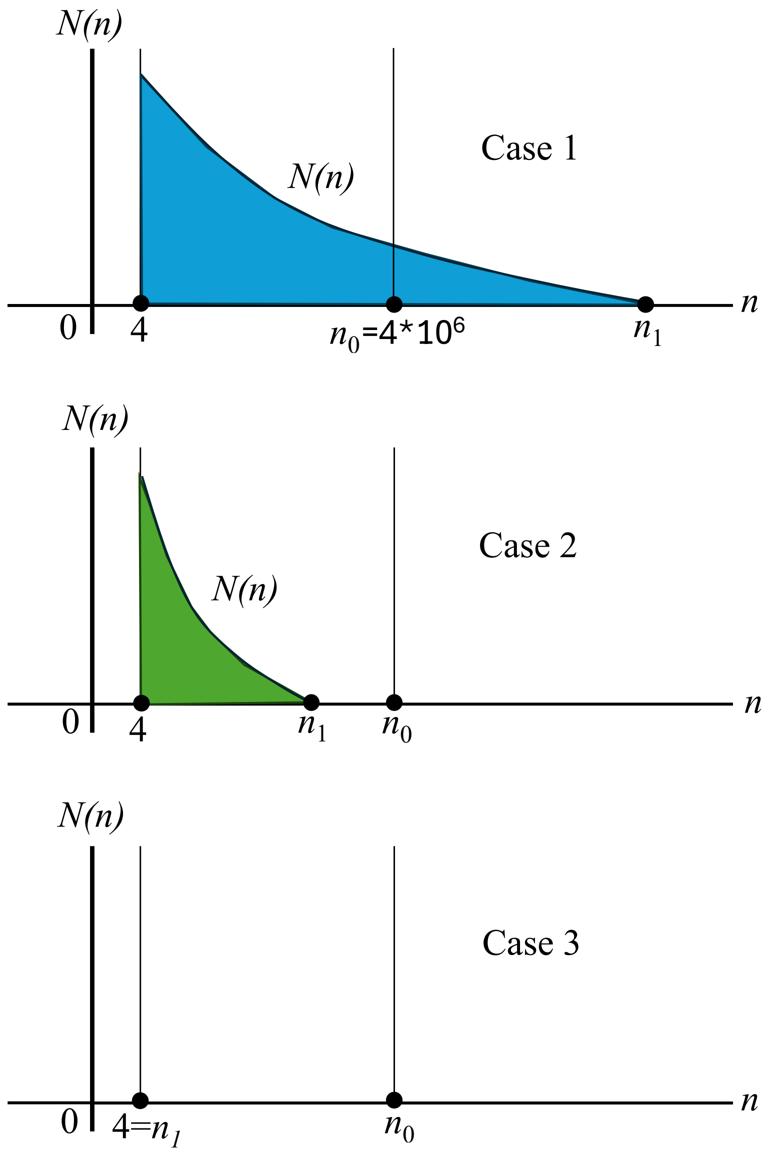

Figure 1 : Case 1 — 3

Toshihiko Ishiwata
E-mail: toshihiko.ishiwata@gmail.com